VERTICALLY DROPPING FREE JET OF NONLINEARLY VISCOUS AND VISCOELASTIC FLUID

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Features of the dynamics of a vertically dropping thick jet of non-Newtonian fluids are investigated.

Axisymmetric fluid flow is described in a long-wave approximation, when the extent of the solid part of the jet considerably exceeds its radius, by the following system of equations [1, 2]

$$\rho\pi R^2\left(\frac{\partial U}{\partial t}+U\frac{\partial U}{\partial z}\right)=\frac{\partial\sigma\pi R^2}{\partial z}+\pi\varkappa\frac{\partial R}{\partial z}+\rho g\pi R^2, \tag{1}$$

$$\frac{\partial\pi R^2}{\partial t}+\frac{\partial\pi R^2 U}{\partial z}=0. \tag{2}$$

The OZ axis is directed along the jet and $\sigma = \hat{T}_{zz} - (\hat{T}_{xx} + \hat{T}_{yy})/2$. Each jet element is considered subjected to uniaxial drawing at the elongation velocity $\partial U/\partial z$ in the computation of the stress tensor. The boundary conditions to system (1), (2)

$$U|_{z=0}=U_0,\ R|_{z=0}=R_0 \tag{3}$$

are posed in a section behind the zone of velocity profile adjustment after emergence from the nozzle, where the flow is substantially nonuniform. For a jet with a low Reynolds number, the size of this zone is on the order of the jet radius. The quantity $\pi R_0^2 U_0$ determines the fluid mass flow rate constant.

In [3] we studied the phase protrait of stationary system (1), (2) for a nonlinearly viscous fluid

$$\frac{\partial U}{\partial z}=\Phi(|\sigma|)\,\mathrm{sign}(\sigma) \tag{4}$$

and a viscoelastic fluid with a model of Maxwell type [2]

$$\sigma+\lambda\left(\frac{\partial\sigma}{\partial t}+U\frac{\partial\sigma}{\partial z}-2\alpha\sigma\frac{\partial U}{\partial z}\right)=3\eta\frac{\partial U}{\partial z};\ \frac{\partial U}{\partial z}>0,\ \sigma+\lambda\left(\frac{\partial\sigma}{\partial t}+U\frac{\partial\sigma}{\partial z}+\alpha\sigma\frac{\partial U}{\partial z}\right)=3\eta\frac{\partial U}{\partial z};\ \frac{\partial U}{\partial z}<0. \tag{5}$$

Here $\Phi(\sigma)$ was not specified for (4), and only the general properties of the flow function were used.

It is shown in [3] that there is a unique solution of the stationary equations (1) and (2) for the nonlinearly viscous fluid (4), for which $\partial U/\partial z \to 0$, $R \to 0$ as $z \to \infty$. The nature of the change in elongation velocity and the longitudinal stress along the jet is determined by the magnitude of a parameter of Froude type $\alpha_V = 2U_0^2/gL_V$, where L_V is the length of a column unit cross section and weight equal to the longitudinal stress for which the elongation rate is $2U_0/L_V = \Phi(\rho g L_V)$. For Newtonian fluids $L_V = (6\eta U_0/\rho g)^{1/2}$, $\alpha_V = (2\rho U_0^3/3g\eta)^{1/2}$. When $\alpha_V \gg 1$ the role of the inertial forces is manifest only far from the nozzle. The elongation rate and longitudinal stress initially grow along the jet, and then diminish. The maximal elongation rate is $(4U_0 g/L_V^2)^{1/3}$. The corresponding value of the longitudinal stress for a Newtonian fluid is $\sim(18\rho\eta^2 g^2)^{1/3}$. For a viscoelastic fluid with the model (5), the solution of the stationary equations (1), (2) for which $\partial U/\partial z \to 0$, $R \to 0$, $\sigma \to 0$ as $z \to \infty$, is unique only for $6\eta/\lambda > 2\rho U_0^2 + \varkappa/R_0$. The manifold of solutions when this inequality is spoiled is due to different values of the longitudinal stress for $z = 0$ and is associated with the presence of "memory"

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in the fluid. It was shown in [3] that the fluid elastic properties appear most clearly for thick jets, when $12\eta/\lambda \ll \rho g^2\lambda^2$, $U_0 < (2\alpha - 1)g\lambda$. In this case the longitudinal stress grows along the jet to reach the maximum $(2\alpha - 1)\rho g^2\lambda^2 \leqslant \sigma_{max} \leqslant \alpha^2\rho g^2\lambda^2$, which considerably exceeds the shear modulus η/λ, and then the stress diminishes. The elongation rate is $\sim 1/2\alpha\lambda$ on the portion of the jet near the section where the maximum of σ is achieved, the elongation velocity is $\sim 1/2\alpha\lambda$. These deductions agree with tests [4]. When $12\eta/\lambda \gg \rho g^2\lambda^2$, the fluid elastic properties do not appear and the results for Maxwell and Newtonian fluids agree.

An analysis performed in [3] showed that for a low exhaust velocity in this stationary jets of nonlinearly viscous and viscoelastic fluids, maximums in the longitudinal stress are achieved only at the nozzle. The magnitude of the maximum is here determined only by the rheological properties of the fluid. If it exceeds the rupture strength of the fluid σ_r then the stationary solid jet does not form. At this time the characteristics of metals and highly concentrated polymer solutions have been studied sufficiently well [5]. It is shown that the rupture stress is determined mainly by the magnitude of the reversible strain accumulated by the fluid. The strength has practically not been investigated for slightly viscous polymer solutions. However, it is known that insertion of passive accumulators in such solutions reduces the strength characteristics. Let us examine the nonstationary efflux of a fluid with low rupture strength.

Let us turn to the Lagrange variables $t,\ z \to \xi = \int\limits_z^{L(t)} \pi R^2 dz,\ dL/dt = U(L,\ t)$ in (1) and (2), where L is the coordinate of a fixed liquid section. Then

$$\frac{\partial}{\partial z} = -\pi R^2 \frac{\partial}{\partial \xi},\quad \frac{\partial}{\partial t} = \frac{\partial}{\partial t} + \left(\pi R^2 U|_L + \int\limits_z^{L(t)} \frac{\partial \pi R^2}{\partial t} dz\right)\frac{\partial}{\partial \xi}. \tag{6}$$

Using the relationship (2) in (6), we obtain

$$\frac{\partial}{\partial t} = \frac{\partial}{\partial t} + \pi R^2 U \frac{\partial}{\partial \xi}. \tag{7}$$

Going over to the Lagrange variables in (1) and (2) is accomplished by means of (6) and (7). The fluid inertial can be neglected in (1) for a slow efflux. In the noninertial approximation (1) and (2) are written in Largrange variables in the form

$$-\frac{2}{R}\frac{\partial R}{\partial t} = \frac{\partial U}{\partial z},\quad \frac{\partial}{\partial \xi}(\sigma\pi R^2 + \varkappa\pi R) = \rho g. \tag{8}$$

Lagrange variables were used in [3, 6-9] for the analysis of jets. In [3, 7, 8], the equations (8) were used without gravity ($g = 0$) to analyze the dissociation of a horizontal capillary jet of a rheologically complex fluid. The complex formulas obtained for the system (8) (see [3, 8]) make difficult the analytical investigation of changes in the longitudinal stress along the jet at a fixed time. Consequently, we shall henceforth examine thick jets for which the capillary forces are inessential. Taking into account that $\xi = 0$ for $\sigma = 0$ ($z = L$) at the free end of the jet, we find from the second equation in (8)

$$\sigma = \frac{\rho g \xi}{\pi R^2}. \tag{9}$$

Since $\sigma|_{\xi=0} = 0$, then $\partial U/\partial z|_{\xi=0} = 0$, and according to the first equation in (8), the quantity $R_c = R|_{\xi=0}$ does not vary with time. It equals the radius of the free jet that relaxes after emergence from the nozzle. The turning of the free end of the jet under the effect of capillary forces is not described in the approximation under consideration. Substituting the formulas for the elongation velocity and the longitudinal stress from (8) and (9) into the rheological equation for the fluid, we obtain an equation that describes the change in the jet radius with time in which ξ enters only as a parameter, i.e., the radius of each jet section is developed independently. The initial condition for the radius of a section is imposed at the time of its emergence from the nozzle for $z = 0$ or $\xi = \pi R_0^2 U_0 t$. The quantity R_0 depends on ξ, since the fluid relaxes in the presence of the longitudinal jet $\sigma|_{z=0}$. For $\xi = 0$ we have $R_0 = R_c$. The computation of $R_0(\xi)$ requires the solution of the problem in the transition zone after the nozzle.

For the nonlinearly viscous fluid (4) in the dimensionless quantities $u = R/R_c$, $\varphi = \sigma/\rho g U_c t_c$, $\bar{\xi} = \xi/\pi - R_c^2 U_c t_c$, $\tau = t/t_c$, where t_c is found from the relationship $1/t_c = \Phi(\rho g U_c t_c)$, U_c is the velocity corresponding to the section with radius R_c, i.e., the flow rate divided by πR_c^2, the change in jet radius is described by the equation

$$-\frac{2}{u}\frac{\partial u}{\partial \tau}=F(\varphi),\quad u|_{\tau=\bar{\xi}}=u_0(\bar{\xi}),\quad \varphi=\frac{\bar{\xi}}{u^2}. \tag{10}$$

Here $u_0 = R_0/R_c$, $F=\Phi(\rho g U_c t_c \varphi)/\Phi(\rho g U_c t_c)$. Since $\rho g U_c = \rho g U_c t_c \Phi(\rho g U_c t_c)$, then the quantity $U_c t_c$ grows more slowly than $U_c^{1/2}$ as the rate of efflux increases for a pseudoplastic fluid, and more rapidly than $U_c^{1/2}$ for dilatant fluid. For a Newtonian fluid $t_c = \sqrt{3\eta/\rho g U_c}$, $U_c t_c = \sqrt{3\eta U_c/\rho g}$.

Integrating (10) we obtain

$$\int\limits_{u(\bar{\xi},\tau)/\bar{\xi}^{1/2}}^{u_0(\bar{\xi})/\bar{\xi}^{1/2}} \frac{2dv}{vF(1/v^2)}=\tau-\bar{\xi}. \tag{11}$$

As τ grows the section radius diminishes for fixed $\bar{\xi}$. Using (11), we find

$$\frac{\partial \varphi}{\partial \bar{\xi}}=\frac{1}{u^2}\left(1-\frac{2\bar{\xi}}{u}\frac{\partial u}{\partial \bar{\xi}}\right)=\frac{\varphi F(\varphi)}{\varphi_0 F(\varphi_0)}\left[\frac{d\varphi_0}{d\bar{\xi}}-\varphi_0 F(\varphi_0)\right]. \tag{12}$$

Here $\varphi_0=\bar{\xi}/u_0^2(\bar{\xi})$. The expression in the square brackets in (12) is independent of the time. As $\bar{\xi}$ grows, the value of $u_0(\bar{\xi})$ does not. Consequently, $d\varphi_0/d\bar{\xi}\geqslant 1$ and the function $\varphi_0 F(\varphi_0)$ grows monotonically as $\bar{\xi}$ increases. Since $F(\varphi)\leqslant 1$ or $\varphi\leqslant 1$, then for $\bar{\xi}_m$, the root of the equation $\partial\varphi/\partial\bar{\xi}=0$, $\varphi_0(\bar{\xi}_m)\geqslant 1$. In the majority of cases, the change in $R_0(\xi)$ can be neglected for filled polymers. Then $u_0 = 1$ and it follows at once from (12) that there is a unique value $\bar{\xi}_m = 1$. The analysis performed shows that for $t \leqslant t_c\bar{\xi}_m$ the longitudinal stress is reduced monotonically along the jet $\sigma|_{t=0}=\rho g U_c t_c \varphi(\tau)$. For $t > t_c\bar{\xi}_m$ the stress increases along the jet up to the section $\xi_m = \pi R_c^2 U_c \bar{\xi}_m$, where it reaches the maximum $\sigma_{max} = \rho g U_c t_c \bar{\xi}_m/u^2(\bar{\xi}_m,\tau)$, and then diminishes. The value of σ_{max} grows without limit with time. When it exceeds the rupture strength of the fluid, the jet is discontinuous at this site. The volume of the fluid portion being separated is $V = \pi R_c^2 U_c t_c \bar{\xi}_m$. The expression $t_p = t_c\tau_p$ can be used to estimate the time of rupture, where τ_p is determined from (11) for $\bar{\xi} = \bar{\xi}_m$, $u = 0$. For $R_0(\xi) \approx R_c$ $\bar{\xi}_m = 1$. As is seen from (12), the section of least radius ($\partial u/\partial\bar{\xi} = 0$) is found below the rupture section. Its coordinate varies with time. As $t \to t_p$ it tends to the section in which the jet is ruptured. The change in jet length in time is given by the relationship

$$L=U_c t_c\int\limits_0^{\tau}\frac{d\bar{\xi}}{u^2(\bar{\xi},\tau)}.$$

As the rate of efflux increases its length grows in proportion to $U_c t_c$ at the time of rupture.

Let us examine the power-law model $\Phi=(\sigma/3^{\frac{n+1}{2}}K)^{1/n}$ and the viscoplastic Shvedov-model $\Phi = (\sigma - \sqrt{3}T_0)/3\eta$ for $\sigma > \sqrt{3}T_0$, $\Phi = 0$ for $\sigma \leqslant \sqrt{3}T_0$ which describe highly filled polymer solutions in greater detail. In the interest of simplification we consider $R_0(\xi) = R_c$. In the case of the power-law model $U_c t_c = 3^{1/2}(KU_c^n/\rho g)^{1/n+1}$, $u = [1 - \bar{\xi}^{1/n}(\tau-\bar{\xi})/n]^{n/2}$, $\varphi=\bar{\xi}/[1-\bar{\xi}^{1/n}(\tau-\bar{\xi})/n]$.

The least value for the radius $u_{min}=[1-(\tau/(n+1))^{1+\frac{1}{n}}]^{n/2}$ occurs in the section $\bar{\xi}_{min} = \tau/(n + 1)$. For $\tau > 1$ the maximal value of the longitudinal stress $\varphi_{max}=1/\left(1-\frac{\tau-1}{n}\right)^n$ corresponds to $\bar{\xi}_m = 1$. For $\tau\to\tau_p=n+1$ $\varphi_{max}\to\infty$. For the viscoplastic model $\rho g U_c t_c = \sqrt{3}T_0/s$, where $s = 1/(1/2 + \sqrt{1/4 + \eta\rho g U_c/T_0^2})$; $u = 1$ for $\bar{\xi}\leqslant s$: $u=\left\{\bar{\xi}/s-(\bar{\xi}/s-1)\exp\left[\frac{s(\tau-\bar{\xi})}{1-s}\right]\right\}^{1/2}$ for $\bar{\xi} > s$. These formulas show that there is a section of length $\sqrt{3}T_0/\rho g$ with radius R_0 near the free end of a jet of viscoplastic fluid where there is no flow. For $\tau > 1$ the maximal value of the stress

$$\varphi_{max}=\frac{s}{1-(1-s)\exp\left[\frac{s(\tau-1)}{1-s}\right]}$$

corresponds to $\bar{\xi}_m = 1$. For $\tau\to\tau_p=1-\frac{1-s}{s}\ln(1-s)\varphi_{max}\to\infty$. Then $\eta\rho g U_c \gg T_0^2$, and the viscoplastic and Newtonial jet formulas agree.

For the viscoelastic fluid (5) the change in jet radius with time is given in the dimensionless variables $u = R/R_c$, $\tau = t/\lambda$, $\bar{\xi} = \xi/\pi R_c^2 U_c \lambda$, $\varphi = \sigma/\rho g U_c \lambda$ by the relationship

$$(4\alpha - 2)\ln\frac{u_0}{u} + \frac{\omega}{2\bar{\xi}}(u_0^2 - u^2) = \tau - \bar{\xi}, \tag{13}$$

Where $u_0 = R_0/R_c$, $\omega = 6\eta/\rho g U_c \lambda^2$. The parameter ω characterizes the relationship between the shear modulus of the fluid η/λ and the weight column of unit section of length $U_c\lambda$. Using (13), we have

$$\frac{\partial\varphi}{\partial\bar{\xi}} = \varphi\left\{\frac{(4\alpha-2)\bar{\xi} - 2\bar{\xi}^2 + \omega u_0^2}{(4\alpha-2)\bar{\xi} + \omega u^2} - \frac{[(4\alpha-2)\bar{\xi} + \omega u_0^2]\,2\bar{\xi}}{[(4\alpha-2)\bar{\xi} + \omega u^2]\,u_0}\,\frac{du_0}{d\bar{\xi}}\right\}.$$

As is seen from this expression, the relative stress distribution along the jet is independent of time. If the change in $R_0(\xi)$ can be neglected, then

$$\frac{\partial u}{\partial\bar{\xi}} = u\,\frac{1 - \frac{\omega}{2\bar{\xi}^2}(1-u^2)}{4\alpha - 2 + \omega u^2/\bar{\xi}}, \quad \frac{\partial\varphi}{\partial\bar{\xi}} = \varphi\,\frac{(4\alpha-2)\bar{\xi} - 2\bar{\xi}^2 + \omega}{(4\alpha-2)\bar{\xi} + \omega u^2}.$$

For $\tau < \bar{\xi}_m$, where $\bar{\xi}_m = \alpha - 1/2 + \sqrt{(\alpha - 1/2)^2 + \omega/2}$, the stress along the jet is reduced. For $\tau > \bar{\xi}_m$ it grows up to the section $\bar{\xi}_m$ where $\varphi_{max} = \bar{\xi}_m/u_m^2(\tau)$ and then diminishes. The value of $u_m(\tau)$ is determined from the relationship

$$-(2\alpha - 1)\ln u_m^2 + \frac{\omega}{2\bar{\xi}_m}(1 - u_m^2) = \tau - \bar{\xi}_m. \tag{14}$$

The least value of the jet radius $u_{min} = (1 - 2\bar{\xi}_{min}/\omega)^{1/2}$ corresponds to the coordinate $\bar{\xi}_{min}$ that is found from the equation

$$-(2\alpha - 1)\ln\left(1 - \frac{2\bar{\xi}_{min}^2}{\omega}\right) = \tau - 2\bar{\xi}_{min}. \tag{15}$$

As $\tau \to \infty$ $\varphi_{max} \to \infty$. The volume of the portion of fluid being separated is $V = \pi R_c^2 U_c \lambda \bar{\xi}_m$.

Let us examine the limit cases of small $\omega \ll 1$ and large $\omega \gg 1$ viscosities. When $\omega \ll (2\alpha - 1)^2$, $\bar{\xi}_m \approx 2\alpha - 1$. Neglecting the second term in the left side of (14), we find

$$\sigma_{max} \approx \rho g U_c \lambda (2\alpha - 1)\exp\left[\frac{t}{(2\alpha-1)\lambda} - 1\right], \quad t \gg (2\alpha - 1)\lambda.$$

In this case the volume being separated is $V \approx (2\alpha - 1)\pi R_c^2 U_c \lambda$. For $\tau \gtrsim (2\alpha - 1)$ we have from (15)

$$\bar{\xi}_{min}^2 \approx \frac{\omega}{2}\left[1 - \exp\left(-\frac{t}{(2\alpha-1)\lambda}\right)\right].$$

For $\tau \gg \sqrt{\omega/2}$ the second term in the left side of (13) is negligible for $\bar{\xi} \gg \omega$ and $u \approx \exp[(\bar{\xi} - \tau)/(4\alpha - 2)]$. Therefore, with the exception of a small jet volume near the free end $\xi \gg 6\eta R_c^2\pi/\rho g\lambda$ the fluid is stretched with the constant elongation velocity $-\frac{2}{R}\frac{\partial R}{\partial t} \approx 1/(2\alpha - 1)\lambda$. This result, which is analogous to the kinetics of thinning of a thread during dissociation of a capillary jet of a polymer solution [3, 7, 8], is caused by the elastic properties of the fluid. For $\omega \gg (2\alpha - 1)^2$ $\bar{\xi}_m \approx \sqrt{\omega/2}$. For $t \to \infty$ $\bar{\xi}_{min} \to \sqrt{\omega/2} \approx \bar{\xi}_m$. The volume of fluid being separated is $V \approx \pi R_c^2 U_c t_0$, $t_0 = \sqrt{3\eta/\rho g U_c}$. For $t \lesssim 2t_0$ we find the first term in the left side of (14)

$$\sigma_{max} \approx \left[3\eta\rho g U_c \Big/ \left(1 - \frac{t - t_0}{t_0}\right)\right]^{1/2}.$$

The first term is significant only for $t \gg 2t_0$. Then the rate of elongation in the section $\bar{\xi}_m$ becomes $1/(2\alpha - 1)\lambda$, and the stress is

$$\sigma_{max} \approx (3\eta\rho g U_c)^{1/2}\exp\left[\frac{t - 2t_0}{(2\alpha-1)\lambda}\right].$$

The elastic properties of the fluid change the jet behavior for $\omega \ll 1$. In this case, the volume of the fluid portion being separated is proportional to the velocity of efflux.

Rupture occurs at the site located considerably above the section with minimal radius $\bar{\xi}_m/\bar{\xi}_{min} \simeq (2\alpha - 1)\sqrt{2/\omega} >> 1$. This is different principally from the behavior of the nonlinearly viscous fluid for which the volume being separated grows more slowly than the first power of the velocity. Thus, for the power-law model the volume is proportional to $U_c^{n/(n+1)}$. An analogy occurs only for n >> 1. At the time of separation the section with minimal radius in the nonlinearly viscous fluid approaches the rupture site from below. When $\omega >> 1$, the behavior of the viscoelastic fluid is similar to that of a viscous fluid.

The results obtained for $\omega << 1$ are in agreement with the experimental data in [10] for slightly viscous suspensions on a polymer base that possess considerable elastic properties. The deductions for the nonlinearly viscous and viscoplastic fluids correspond to the tests in [11] for a highly filled suspension on a polymer base with high viscosity.

The character described above for the jet rupture is realized for low velocities when the stress near the nozzle does not exceed σ_{lim} in the first stage of the efflux ($\tau \leqslant \bar{\xi}_m$). Otherwise the rupture occurs near the nozzle and the volume of the fluid being separated is $V \simeq \pi R_c^2 \sigma_{lim}/\rho g$.

As the rate of efflux increase, the role of the fluid inertial forces increases. An analysis [3] of the stationary equations (1) and (2) with the inertial forces taken into account shows that the maximal value of the longitudinal stress is reduced with the growth of velocity. When this quantity becomes less than the rupture strength of the fluid, a stationary dropping jet is formed.

NOTATION

ρ, fluid density; R, jet radius; U, fluid velocity; $\varkappa$, coefficient of surface tension; g, free-fall acceleration; $\hat{T}_{ii}$, stress tensor components; σ, longitudinal stress; $\Phi(\sigma)$, flow function; λ, relaxation time; α, a parameter; $0.5 < \alpha \leqslant 1$; η, viscosity; T_0, yield stress of the Shvedov–Bingham model for shear flow.

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